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TECHNICAL REPORT
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DESIGNING STRUCTURES FROM FABRICS,
INDUSTRIAL FABRICS FROM STRUCTURAL STRESSES TO FABRIC,
YARN AND FIBER STRENGTH



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April 1975

UNITED STATES ARMY
NATICK RESEARCH and DEVELOPMENT COMMAND
NATICK, MASSACHUSETTS 01760



Aero-Mechanical Engineering Laboratory

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20. Abstract (cont'd)

to other structures, such as parachutes, pressure suits, or conveyor belts, for which structural stresses can be determined. The start of the analysis is to determine the external imposed stresses on the structure and to relate this to fabric loads in the warp and filling directions. Theoretical and experimental approaches using methods of applied mechanics are then used to relate the design of the fabric to the exact mechanical requirements of the structure. The effect of the geometric shape of the structure on the fabric design is discussed.

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Preface

This report was prepared by the Aero-Mechanical Engineering Laboratory, (AMEL), Engineering Science Division (ESD). The report summarizes some work accomplished by this Laboratory over the past ten years under the general heading of the Structural Mechanics of Tentage. The work done to determine the loads on pneumatic shelters was accomplished under contract with the Hayes International Corporation, Birmingham, Alabama, under Technical Effort AE005BG. The work on the biaxial stress-strain properties was accomplished under contract with the Fabric Research Laboratories, Dedham, Massachusetts under Technical Effort AE002BG. The work on the theoretical prediction of yarn strength was accomplished at the University of Manchester Institute of Science and Technology, Manchester, England, by the author under a Research and Study Fellowship given to the author by the Secretary of the Army. All the work, contracts and Fellowship effort, was carried out under the direction of Dr. Constantin J. Monego of the US Army Research and Development Command (NARADCOM). NARADCOM has in the past been known as the US Army Natick Development Center (NDC) and the US Army Natick Laboratories (NLABS).

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DESIGNING STRUCTURES FROM FABRICS

Industrial Fabrics from Structural Stresses
to Fabric, Yarn and Fiber Strength

by

C. J. Monego

INTRODUCTION

In keeping with the philosophy of the "total picture", namely, the design of a fabric specifically for its end use, the fabric industry can no longer afford to sit still and be complacent, in the traditional way, and design fabrics and let the consumer find uses for them. This route is too time-consuming and costly. It also is nonproductive in defining to the fiber manufacturers what the textile industry needs in the way of new fibers to satisfy the consumer market. This topic is so broad in scope that it requires narrowing down the discussion to a specific area of textile application. In this paper the specific area of the application of fabrics will be confined to industrial fabrics. These are fabrics which are used structurally to resist external forces in such applications as pneumatic structures, tire cords, conveyor belts, parachutes, pressure suits and, thinking in the future, even structures for outer space. Textiles offer engineers and architects an excellent building material because of their high strength-to-weight ratio, the strength of textile material on the traditional pound-per-square-inch basis is equal to that of the best steels on an equal weight basis.¹ They have an additional advantage in that they do not suffer from fatigue under normal structural loads and can stand many bending and vibration cycles without failure while supporting the loads imposed on the structure.

On the other hand, all textiles have relatively poorer sunlight and weathering resistance than steel. For example, among all textile fibers nylon has a relatively poor but commercially acceptable sunlight and weather resistance; polyesters are significantly better in this respect than the nylons.

The sunlight and weather resistance of nylon and polyester fibers has been greatly improved by the application of finishes to the fabric. One method is to coat the fabric with elastomeric materials; another is to treat the fabric with sunlight inhibitors.

¹ Kaswell, E. R., Welling Sears Handbook of Industrial Textiles, 1963, Wellington Sears Company, Inc., NYC, NY.

While much has been done to significantly improve the sunlight and weather resistance of nylon, polyester and other fibers, more work needs to be done to further improve their weathering resistance characteristics.

An additional factor to consider when thinking of the use of textiles as a building material is that textiles will not support compression. Therefore, textiles are most applicable in tension structures. This is not a great limitation because building in tension is being considered by leading architects and engineers for the future, for example, Frei Otto's work on fabric and thin film structures.^{2,3} Building in tension is a means of producing lightweight construction, and its advantages over the traditional engineering practices of the past is illustrated by the comparison of the wagon wheel which carries load in compression, and a bicycle wheel which carries load in tension. From this comparison, it is clear that the difference between current conventional buildings, built in compression, and the structures contemplated in the future, built in tension, is one of weight and the amount of material required. Since textiles are ideally suited for use in tension structures, the textile industry should capitalize on this trend in construction to expand its production of industrial fabrics. With the construction industry talking of constructing multi-story structures in tension,⁴ a large surface area must be covered and textiles are likely candidates for this type of application. This demand would mean a large market for textile fabrics heretofore not existing.

To tap this potential market, the textile industry must convert the art of making textiles into the engineering design of textile fabrics. To do this within the theme of the "total picture", one must start with the structure in which the textile material will be used. The stresses and strains in the structure resulting from service loads must be determined and these stresses used to design the fabric to optimally meet the need of the desired structure.

²Otto, Frie, *Tensile Structures, Volume One, Pneumatic Structures*, 1967, The MIT Press, Massachusetts Institute of Technology, Cambridge, Massachusetts, and London, England.

³Otto, Frie, *Tensile Structures, Volume Two, Cables Nets and Membranes*, 1969, The MIT Press, Massachusetts Institute of Technology, Cambridge, Massachusetts, and London, England.

⁴Pohl, J. G., *Architecture in Australia*, Vol 4, 1968, p 635.

To bring to the attention of engineers and architects the potential for use of textile fabrics as a construction material and thus to encourage the growth of this market, the textile industry must address itself to providing the necessary engineering data for these materials. In addition, more effective design approaches must be developed.

The objective of this paper is to illustrate how this "total picture" philosophy of the design of industrial fabrics can be applied by reviewing the program currently underway at the US Army Natick Research and Development Command, Natick, Massachusetts, on the design of pneumatic structures. The review will include statements concerning the current state of the art in the mathematical modeling and experimental knowledge of pneumatic structures, fabric stress-strain behavior, and yarn load-elongation behavior. In addition, the needed improvements in the state of the art in each of these areas are discussed. Following this is a discussion of a plan for the optimum design of pneumatic structures based on the design data obtained for the structure fabric and yarn.

This plan is to develop a procedure and associated computer program to predict the mechanical response of fibers, yarn, fabric, and the structural response to externally imposed loads. This computer program will insure the development of structures of minimum weight and, as a consequence, minimum cost.

Development of such a design approach for a structure is necessary. Placing the end item characteristics, structural loads, fabric geometry, yarn structure and fiber stress-strain characteristics in a coordinated computer program would enable a textile engineer to explore a number of structural designs in a few minutes and to obtain complete specifications for the fabrics, yarns and fibers without tracing a line on a piece of paper.

PNEUMATIC STRUCTURES

At the present time pneumatic shelters and the fabrics they are made from are engineered independently. The shelters manufacturer designs the shelters using various mechanical analysis procedures and determines the stresses in their fabric shell. The shelters manufacturer then selects a fabric, engineered by a textile firm, which most nearly meets his shelter requirements. With this procedure either one of two things is likely to happen with a high frequency: one, a fabric which is underengineered for the shelter is selected, this will lead to premature failure or a shortened service life for the shelter; two, a fabric may be selected which is overengineered for the shelter, this leads to increased costs and excessive weight for the shelter. Clearly then, the present procedure for fabricating pneumatic shelters will not effectively lead to shelters of the lowest cost and weight while meeting the desired requirements.

What is needed is a unified engineering basis for designing pneumatic shelters and for determining the stresses in the fabric. The fabric stresses so determined should then be the basis for engineering a fabric to optimally meet these stresses. Only by engineering a fabric to meet the fabric stress conditions in the structure can an economical, reliable, and efficient pneumatic shelter be obtained.

The following will discuss one approach taken in this direction. It will describe the present state of the art for designing pneumatic structures including experimental and theoretical procedures. In addition needed improvements in the state of the art and a summary of what was accomplished by this work will be discussed.

Present State of the Art

The starting point for the design of any fabric for minimum weight and maximum strength is a consideration of the structure in which it is to be used. The condition which must be fulfilled in this approach is that one must be able to specify the stresses in the structure so that these can be used in the design of the fabric and the yarn utilized in the fabric. To accomplish this for pneumatic structures loading due to wind has been determined experimentally and this loading used in an analysis to predict the stress in the shelter. Scale models of spherical and cylindrical pneumatic structures were constructed and subjected to wind tunnel tests to determine the loads resulting from wind. The description of these tests and results are presented in Dietz et al.⁵ Figures 1 and 2 depict two typical full-scale pneumatic shelters which the scale models represent.

⁵Dietz, A. E., Proffitt, R. B., Cabbot, R. S., Moak, E. L., Design Manual for Ground Mounted Air-Supported Structures (Single and Double-Wall), US Army Natick Laboratories, Natick, Massachusetts, 1969, Technical Report 69-59-GP.

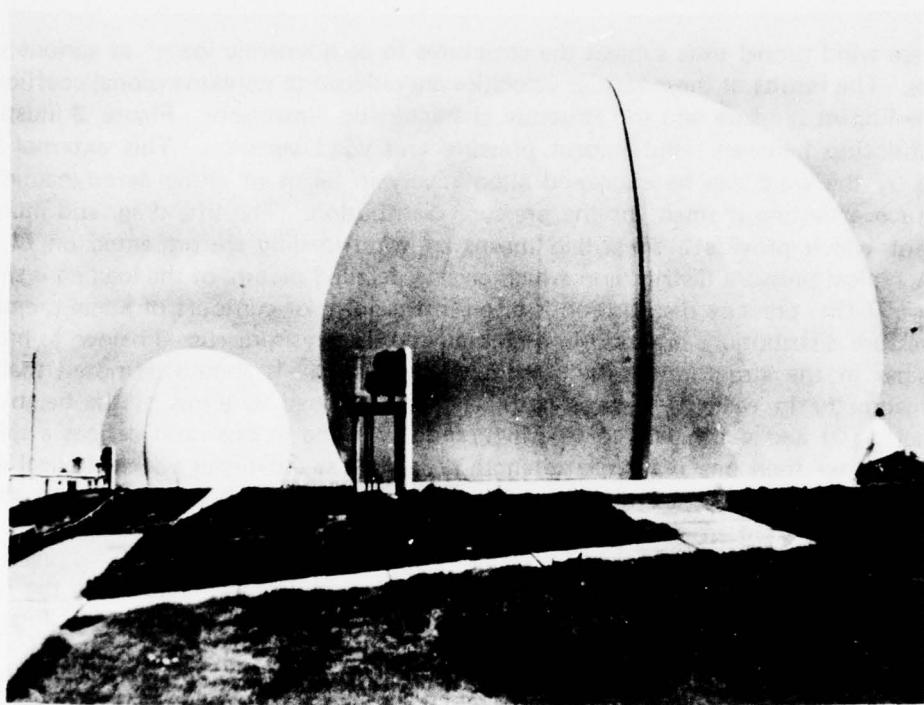


Fig.1. Single-Wall Air-Supported Tent. (Sphere)

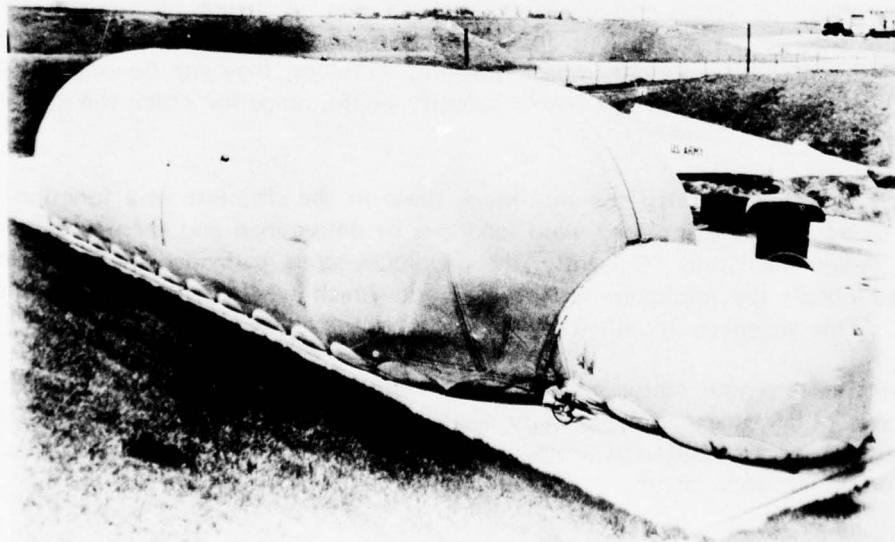


Fig.2. Single-Wall, Air-Supported Tent. (Cylinder)

These wind tunnel tests subject the structures to aerodynamic loads⁵ at various wind velocities. The results at these various velocities are reduced to nondimensional coefficients using the impact pressure and the structure characteristic dimensions. Figure 3 illustrates the relationship between wind impact pressure and wind velocity. This external load imposed by the wind can be expressed alternatively in terms of either aerodynamic lift, drag, and overturning moment or the pressure distribution. The lift, drag, and moment coefficient which provide a simplified means of wind loading are presented on Figures 4-6. A typical pressure distribution which gives a detailed picture of the loading is shown on Figure 7 (the pressure distribution is given in the form of contours of equal pressure). Such pressure distributions are used in a mathematical analysis, discussed below, to predict the stresses in the structure resulting from wind loading. It should be noted that the shelter geometry in each of Figures 4, 5, and 6 is defined in terms of the height (H) to diameter (D) and width (W) to length (ℓ) ratios. A one to one ratio defines a sphere, a one to greater than one diameter to length ratio defines a cylinder with spherical ends. The anchor loads, which were measured separately and for which coefficients are shown on Figure 8, are used in the design of the structure anchoring system.

Mathematical Modeling: The fabric stress was analytically determined using the linear membrane theory and the pressure distribution obtained from model tests Figure 7 (sphere). The results found for maximum fabric stress are shown on Figure 9. This shows the fabric stress for a shelter with a height-to-diameter ratio of one-to-two, or one-half the height to the diameter. Similar curves are generated for three-eighths and three-quarters height-to-diameter ratios, and extrapolation between the given values may be made.

Figure 9 shows the meridional filling stress coefficient for cylindrical tents with hemispherical ends. Figure 10 shows the relationship among maximum meridional (filling) and circumferential (warp) and shear stress coefficients for spheres, and Figure 11 shows maximum shear stress coefficient for cylinders. These coefficients have been made nondimensional using the wind impact pressure, therefore, they can be used to calculate the actual stress for any specified wind velocity in the range for which the data is valid, up to 50 m/s.

With the above results, the maximum stress in the structure as a function of size of the shelter and the imposed wind load can be determined and these stresses can be used to design the fabric. Currently, the maximum stress is divided by a fiber strength factor to obtain the minimum weight of fabric which would withstand the structural stresses. This weight is modified by an appropriate safety factor.

The fiber strength factor which is the fabric breaking load divided by the fabric weight was obtained experimentally by averaging at least ten breaking strength tests on plain weave fabrics of different weights. Fiber strength factors were found for six fibers; nylon, polyester, spun acrylic, glass, and polypropylene.

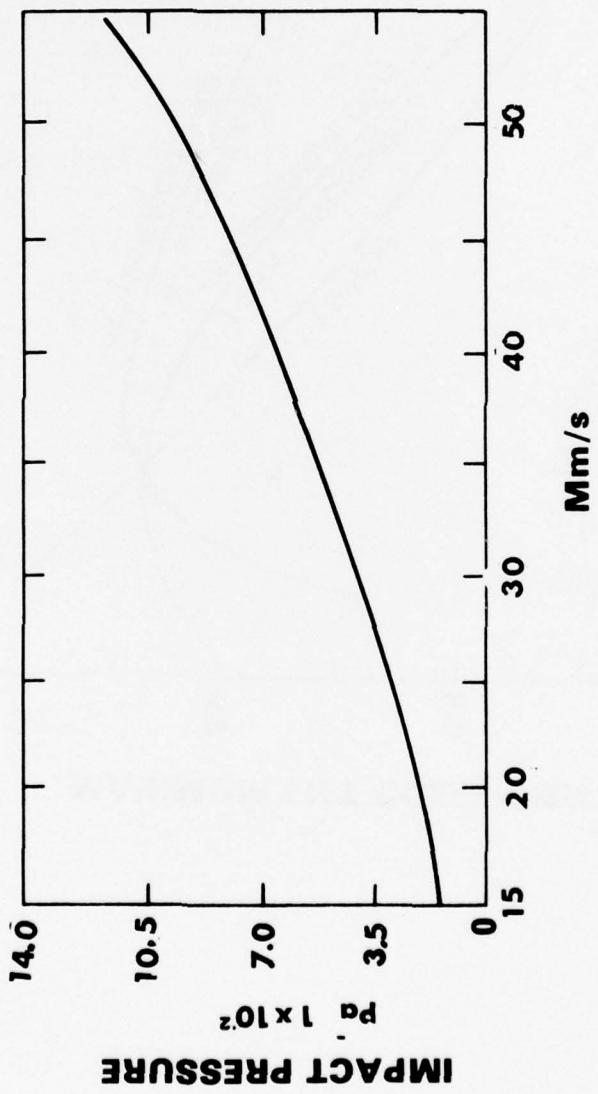


FIG. 3 VARIATION OF IMPACT PRESSURE WITH AIR SPEED AT SEA LEVEL; STANDARD ATMOSPHERE

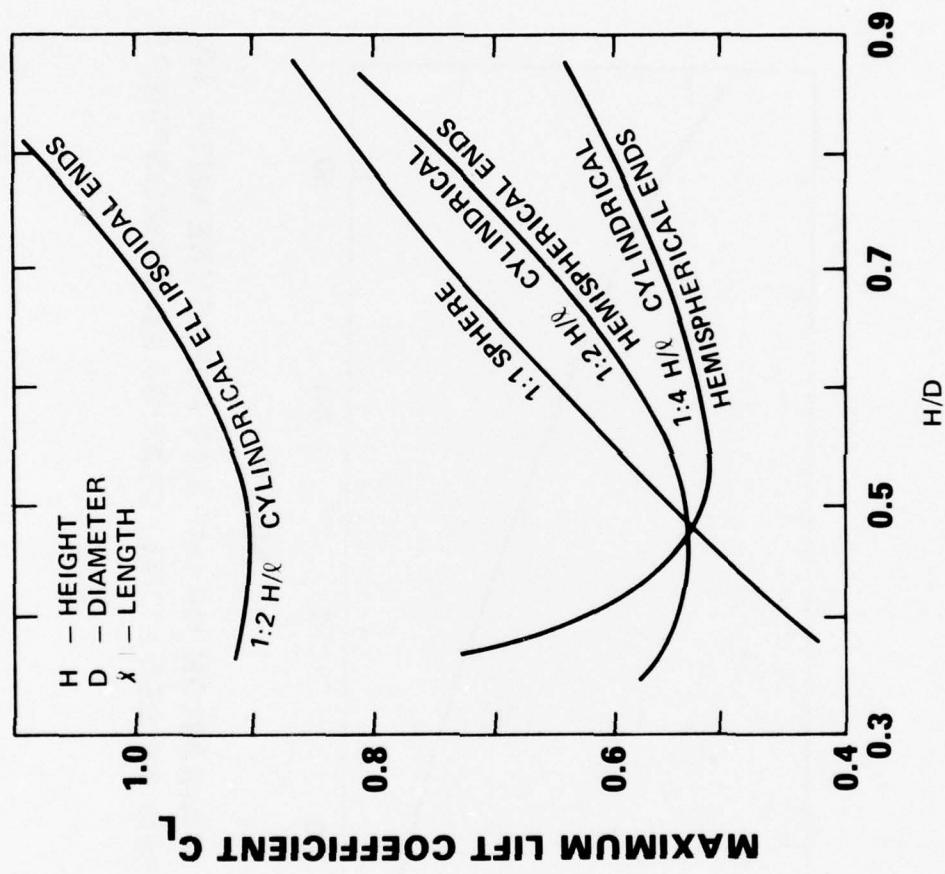


FIG. 4 VARIATION OF LIFT COEFFICIENT WITH SHAPE

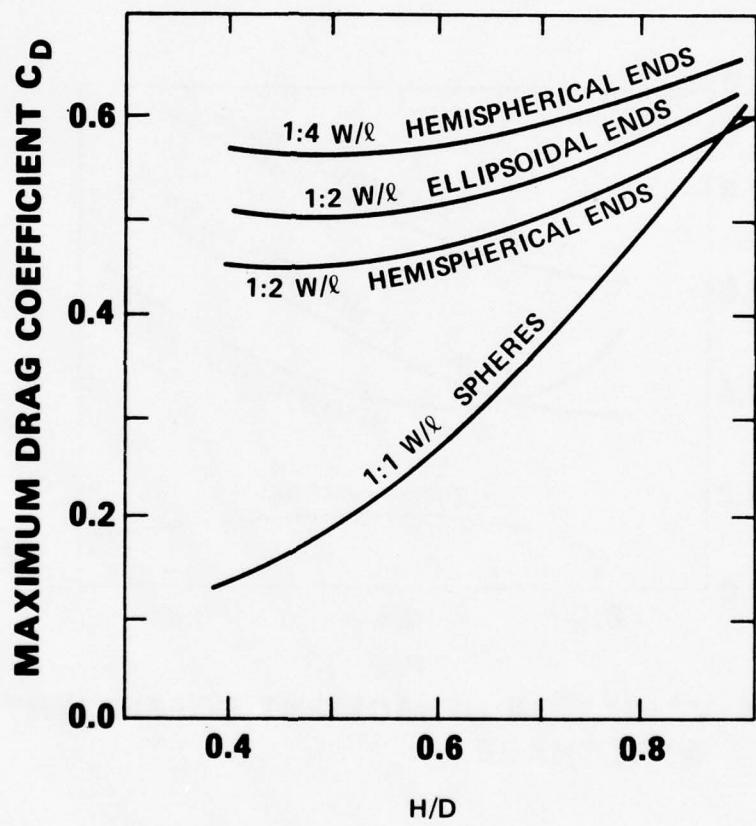


FIG. 5 VARIATION OF DRAG COEFFICIENT WITH SHAPE

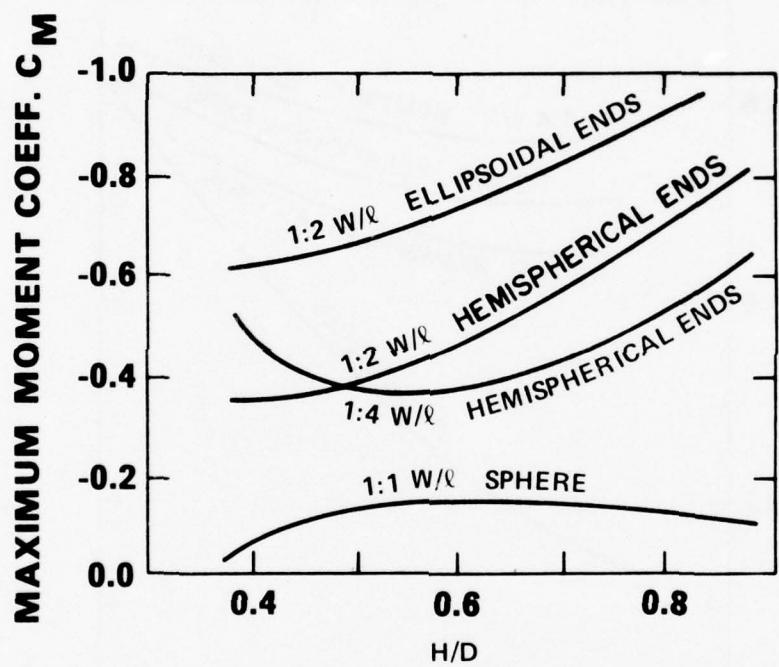
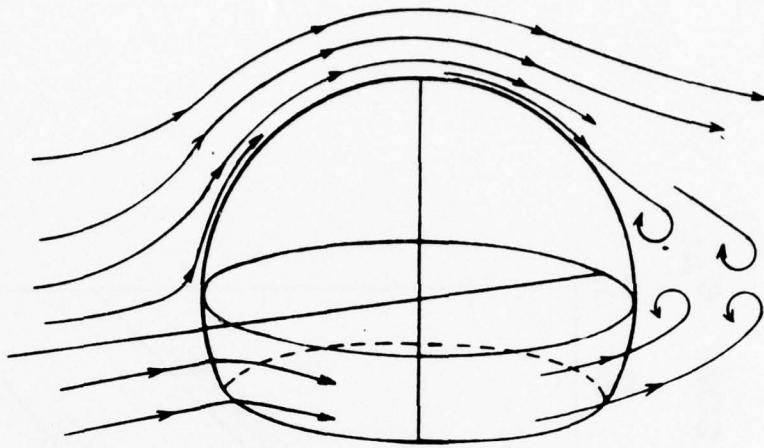


FIG. 6 VARIATION OF MOMENT COEFFICIENT WITH SHAPE



Typical Air Flow

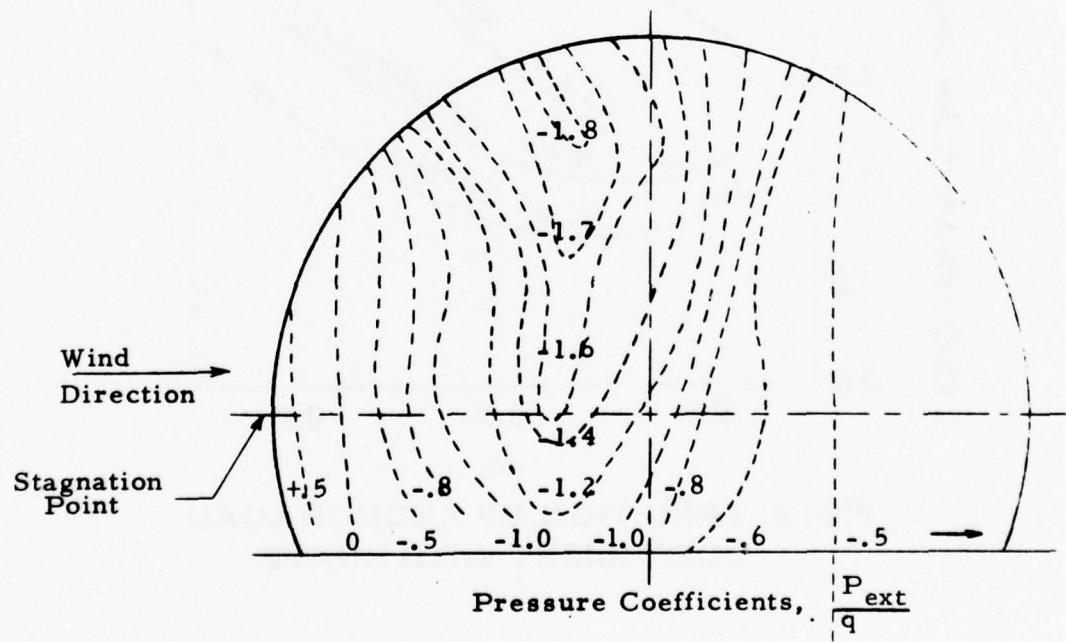


Fig. 7 Typical Pressure Distribution on a Spherical Shell

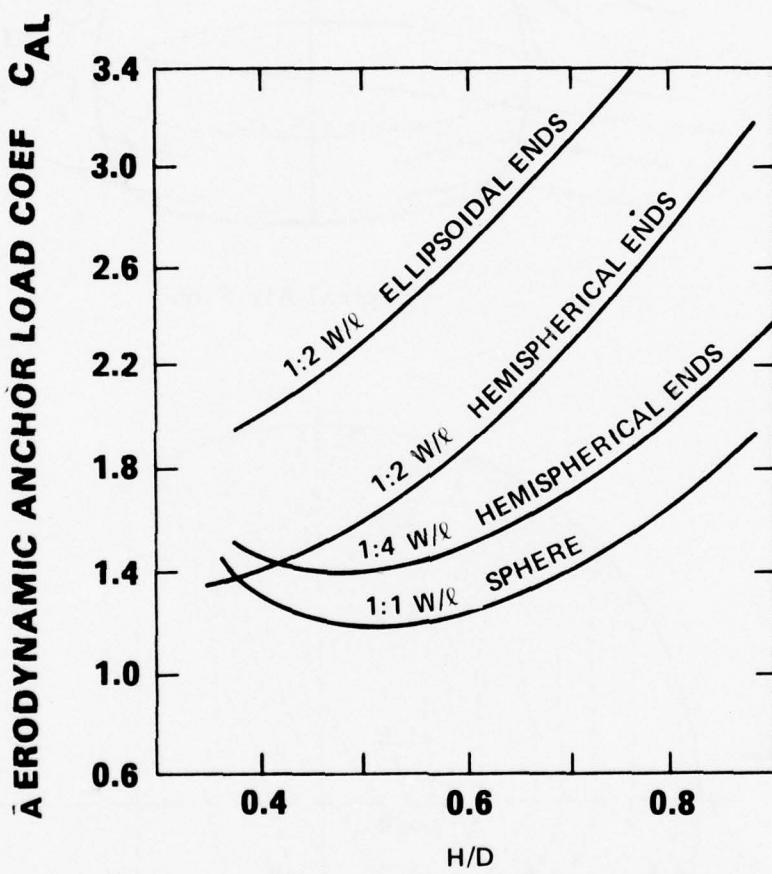


FIG. 8 VARIATION OF ANCHOR LOAD COEFFICIENT WITH SHAPE

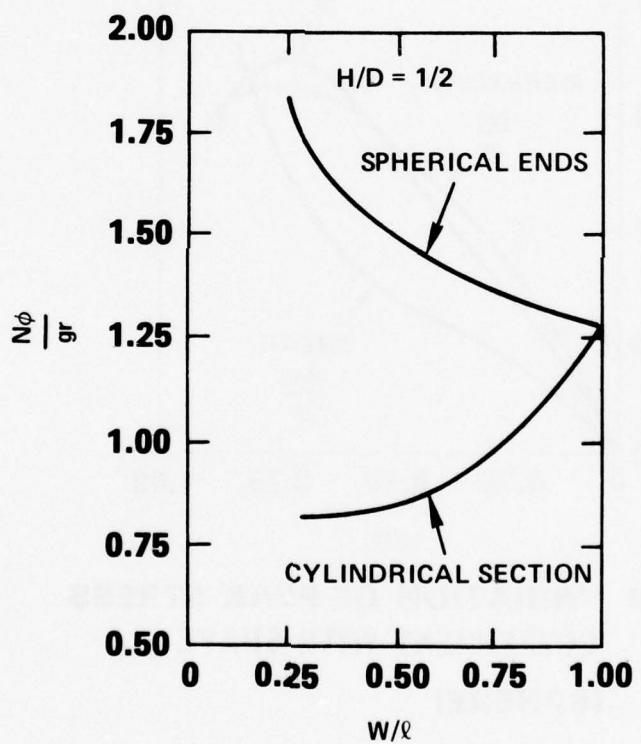


FIG. 9 VARIATIONS OF MAXIMUM DESIGN STRESS COEFFICIENT WITH TENT WIDTH TO LENGTH RATIO.

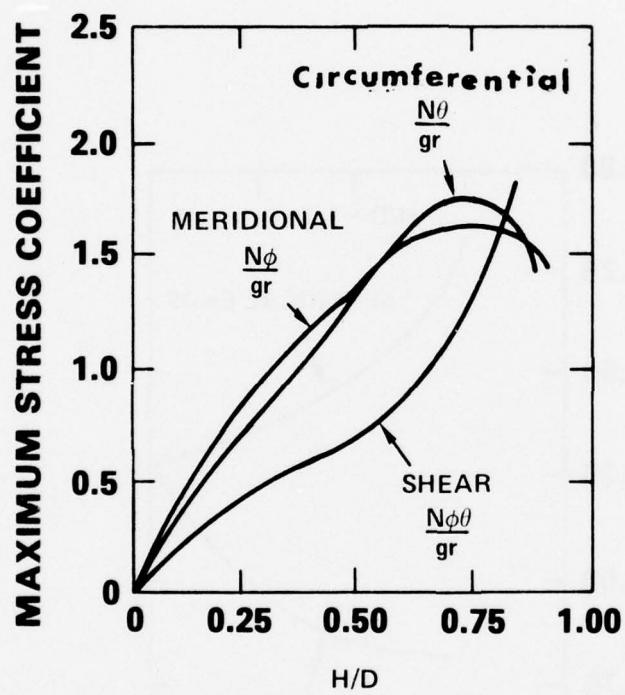


FIG. 10 VARIATION OF PEAK STRESS COEFFICIENT WITH SHAPE (SPHERE)

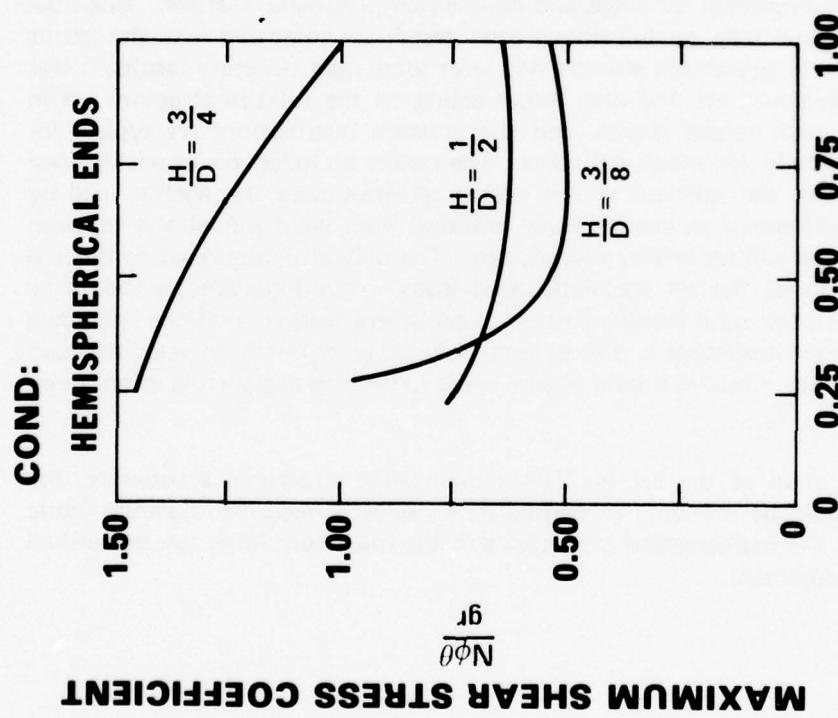


FIG. 11 VARIATION OF MAXIMUM DESIGN STRESS COEFFICIENT WITH TENT WIDTH TO LENGTH RATIO.

This technique for determining and selecting fabric weight required to withstand the maximum stress is rather crude and is based on uniaxial loading test results when biaxial test results are required. A more thorough understanding of the biaxial stress-strain behavior of fabrics is needed so that biaxial results can be used to determine fabric weight and other parameters. A program to accomplish this is discussed below.

Needed Improvements in the State of the Art

The present state of the art includes: engineering data for four different shapes of pneumatic structures; i.e. spheres, cylinders with spherical ends, cylinders with ellipsoidal ends, and double-wall cylinders with flat ends. Additional work is needed to provide engineering data for pneumatic shelters of shapes other than those described in this section. In addition, all of the engineering data for pneumatic shelters was obtained on reduced size, scale model shelters. The available data had to be verified by testing full-size tents.

This has been accomplished for single-and double-wall pneumatic shelters. When the results of the wind tunnel tests on full-scale shelter tests⁶ are compared with the results obtained using models of pneumatic shelters and with some rigid structure results, it was found that the aerodynamic lift and drag forces acting on the full-size structure are in reasonable agreement with model results, and the pressure distributions are typical for those found experimentally for rough cylinders. The results on independent anchor load measurements show that the previous model results underestimate the anchor load by a factor of 2. This difference in anchor loads resulting from wind tunnel test between model and full-size tents will be further investigated. The only structural load considered under the present state of the art are static wind loads. Additional studies should be conducted to determine the wind loading pattern under severe storm conditions (including hurricanes, typhoons, and tornados) and to determine whether or not wind velocities have an accurate gust frequency. Studies should also be made to develop engineering information on static snow loads.

To improve the state of the art for determining fabric stresses in a structure, the testing techniques should be refined. In particular, a device is needed to measure fabric loads directly so that the mathematical predictions of the maximum loads can be verified by experimental measurement.

⁶Madden, R., Wright, H. A., Murray, B. E., Clemente, A. R., Blackwell, J. D., Measurement of Wind Loads on Large-Scale Air-Supported Shelters, Bolt Beranek and Newman, Cambridge, MA and the US Army Natick Research and Development Command, Natick Technical Report 75-101 AMEL, 1975.

Summary

To summarize the work discussed in this section, engineering data have been obtained which can be used to design and construct single-wall pneumatic shelters of the following shapes: spheres, cylinders with spherical ends, cylinders with ellipsoidal ends, and although not discussed, double-wall cylinders with flat ends. A computer program is available which will enable an engineer, who provides as input to the program the shelter shape, size, and wind speed, to obtain the aerodynamic loads on the shelter in terms of lift, drag, and moment; anchor loads; size and power requirements of the inflation system; and the estimated weight and cube of the finished shelter. The program also includes as output the following fabric parameters: maximum fabric stress in the warp and filling directions, the minimum weight of the base fabric required by the shelter, and the estimated coated fabric weight. The data output from this program on stresses in the fabric can be used as input to the engineering design of the fabric once a thorough understanding of the biaxial stress-strain behavior of the fabric is achieved. A program to provide this understanding is discussed in the next section entitled, "Fabric Stress-Strain Characteristics".

FABRIC STRESS-STRAIN CHARACTERISTICS

The results in this section are independent of any structure but relate to the design of the fabric. NARADCOM's purpose is to couple this fabric and yarn data to the design of pneumatic structures, but it is possible to couple this data to the design of many other systems or structures.

The stresses in the structure due to imposed loads provide a measure of the strength of the fabric needed to support these loads and thus serve as criteria for the design of the fabric to effectively design the structure. A thorough knowledge of the biaxial stress-strain behavior of fabrics as a function of their construction variables is necessary. A program to obtain such information was conducted, and a review of this program is presented in this section.

The tensile properties of fabrics are usually given in terms of loads and deformation measured along only one axis. For many applications, this data is adequate; it gives an index of relative strength. However, rarely are truly uniaxial loads imposed on fabrics in the numerous practical applications in which they find use. In most instances, loads are imposed simultaneously in more than one direction; i.e., the deformation at the knee or elbow of a garment, the canopy of a parachute, the walls of pneumatic structures, etc.

The efficient use of fabrics in these structures requires the development of precise design formulae. However, before design formulae and specifications can be written, a better knowledge and understanding is required of the two-dimensional, load-extension characteristics of fabrics than is presently available.

For the complete theoretical derivation of the deformation and stresses in the fabric, the cited references should be reviewed. It is obviously an unending task to make and test experimentally all possible fabric combinations to find the one fabric most suitable for a particular structure. Even if this were done, without computer aid the result might become irretrievably lost in cataloguing for future reference. What is needed is a theoretical deformation analysis of a model fabric which can be used to calculate fabric biaxial stress-strain behavior. A program to develop this theory was supported by the U.S. Army Natick Research and Development Command, Natick, Massachusetts and carried out by the Fabric Research Laboratories, Dedham, Massachusetts, as discussed below.

Present State of the Art

Under biaxial conditions, the stress-strain response of the fabric is strongly dependent on the ratio of the loads in the two directions. This behavior as exhibited by a commercial nylon fabric, is illustrated in Figures 12 and 13. The appropriate ratio depends on the fabric application; in inflated spherically-shaped structures, the nominal loading ratio is 1:1; in inflated cylindrically-shaped structures, it is 2:1. Other structural shapes and loading will impose different ratios of warp to filling loads.

Three test procedures are currently used to measure the biaxial stress-strain behavior of fabrics. These are: the pressurized diaphragm which gives a biaxial stress ratio of 1; the pressurized circular cylinder which gives a biaxial stress ratio of 2,⁷ and the biaxial tester which gives a variable biaxial stress ratio.

Figure 12 shows the curves for the warp direction and Figure 13 represents the curves for the filling direction. The curve designated as A in both figures represents the uniaxial stress-strain response for the fabric. It should be noted that the curve A for the warp, Figure 12, is steeper than that in the filling direction, curve A of Figure 13. Since uniaxial tests represent a straightening of the yarn systems under load, this indicates that the warp yarns have less crimp; hence, they appear stiffer and go sooner into extension under load than the filling yarns and require less extension to straighten the crimp before fiber elongation sets in. Since the uniaxial tests represent primarily a tensile stress of the yarns under an imposed load, this test will be taken as a point of reference for the results of the biaxial tests. Curve B, Figures 12 and 13, represents the results of a pressurized diaphragm biaxial stress-strain test, a 1:1 warp-to-filling loading ratio which is the loading ratio for a sphere. In Figure 12, curve B indicates that the warp is not as stiff at low

⁷ Monego, C. J., The Biaxial Stress-Strain Behavior of Fabrics, US Army Natick Laboratories, Natick, Massachusetts, 1965, Technical Report ME-4.

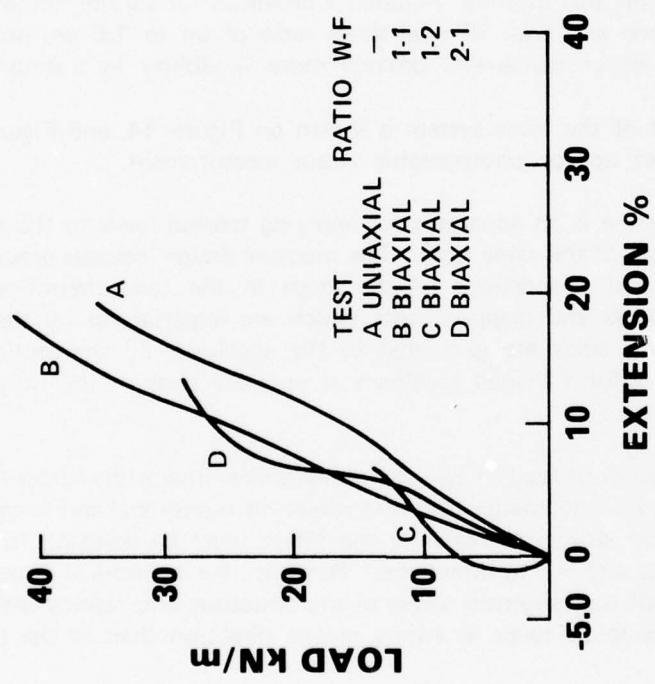


FIG. 12 LOAD EXTENSION RESPONSE
FOR 68 g/m² NYLON FABRIC,
POLYURETHANE COATED
(WARP YARNS)

loads and somewhat stiffer at rupture elongation than the uniaxial case. The greater elongation at low loads of the warp yarns is due to the mechanism of crimp interchange between the warp and filling yarns. This same mechanism shows the filling yarns which have the highest crimp, curve B, Figure 13, to be markedly stiffer than the uniaxial case. The increased stiffness shown by the filling yarns happens because both yarn systems are loaded at the same rate and both yarn systems mutually resist the yarn straightening action imposed by their respective loads. This is a good example of the effect of crimp interchange.

These two test procedures are limited in the magnitude of the stress ratio that can be obtained; to overcome this limitation a biaxial testing machine was designed and constructed. The design of this test machine included a provision for varying the biaxial stress ratio to greater than one and two. Biaxial stress ratio of up to 1:6 are possible on the machine. This is a factor which will provide more flexibility in testing.⁸

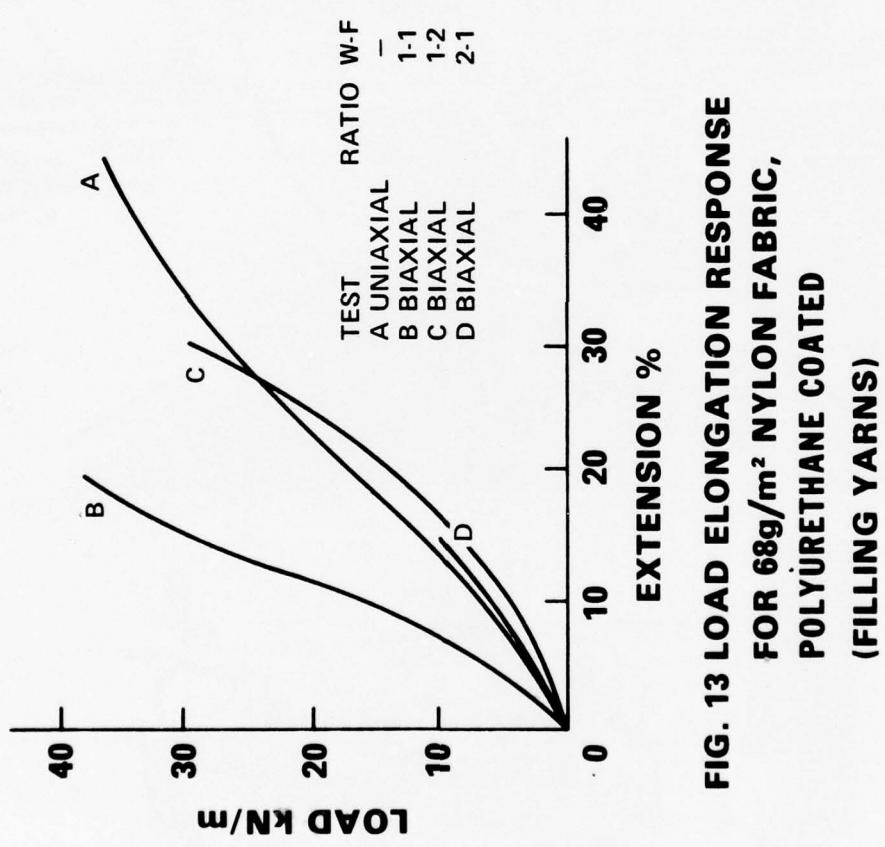
The general arrangement of the force system is shown on Figure 14, and Figure 15 shows the testing machine set up for photographic strain measurement.

This Biaxial Tensile Machine is an apparatus for applying tension loads to the fabric in the warp and filling direction at the same time. The machine design includes provisions for controlling the amount and proportions of the loads in the two directions and instruments for measuring forces and displacements which are experienced by the test specimen. The biaxial tension loads are generated in the specimen by the motion of four jaws which grasp the cruciform shaped specimen at opposite ends of the warp and filling direction.

The simultaneous application of load in two directions approximates the forces acting on the fabric in real situations, i.e. pneumatic structures where the meridional and tangential stresses are equal for spherical structures meaning the fabric must be designed to have the same strength in the warp and filling directions. However, for cylindrical structures the meridional stress is one-half the tangential stress of the structure, and fabrics designed for cylindrical structures have to be twice as strong in one direction than in the other.

Results obtained with the biaxial tester are presented on Figures 17-20 and will be discussed subsequently in connection with the comparison of theoretical and experimental results.

⁸Sebring, R. E., and Freeston, W. D., Biaxial Tensile Tester for Fabrics, Fabric Research Laboratories, Inc., Dedham, Massachusetts, and the US Army Natick Laboratories, Natick, Massachusetts, Phase I, Final Report. Limited distribution.



**FIG. 13 LOAD ELONGATION RESPONSE
FOR 68g/m² NYLON FABRIC,
POLYURETHANE COATED
(FILLING YARNS)**

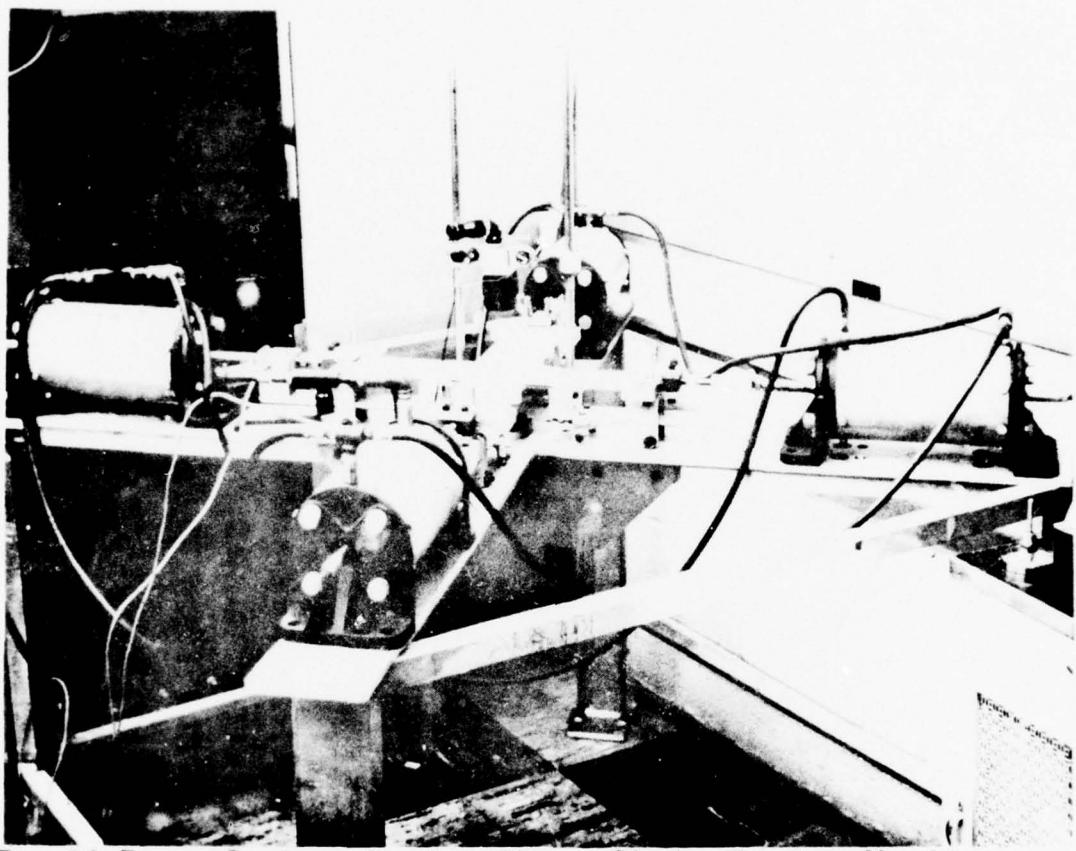


Fig. 14 Force System, Biaxial Stress-Strain Testing Machine

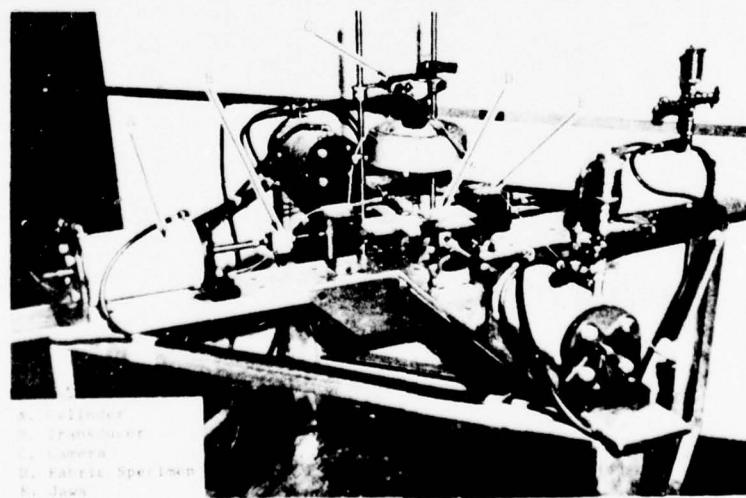


Fig. 15 Biaxial Stress-Strain Testing Machine
Photographic Strain Measurment.

Mathematical Modeling: The theoretical prediction of the stress-strain response of fabrics under two dimensional loading has received the attention of previous investigators,^{9,10,11,12,13,14,15} however, formulas or computer programs which engineers can readily use have yet to be developed. The required computer program needs to be developed if the textile industry wishes to tap the full potential of the rapidly expanding market in mechanical fabrics. Work to develop such a program is discussed in the present section.

The complete deformation analysis of a biaxially-stressed woven fabric involves the use of a large number of parameters and the consideration of many deformation mechanisms. For example, in the model used in reference 7 and 9, eleven parameters are required to define the geometry of a plain-weave fabric and four to express the orthogonal deformation of stresses and strain.¹⁶ The consideration of yarn structure and filament properties necessitates the introduction of even more parameters.

⁹Adams, D. P., Schwarz, E. R., and Backer, S., Text. Res. J., 1956, 26,9.

¹⁰Davidson, D. A., The Mechanical Behavior of Fabrics Subjected to Biaxial Stress, Part 1. Theoretical Analysis of the Plain Weave Fabric, Technical Documentary Report No. ASD-TDR-63-485, USAF, Dayton, Ohio, 1963.

¹¹Reichardt, C. H., Woo, H. R., and Montgomery, D. J., Text. Res. J., 1953, 23, 424.

¹²Popper, P. G., A. Theoretical Investigation of Crimp Interchange in a Woven Fabric Under Biaxial Stress, Technical Documentary Report, ASD-TDR-62-457, USAF, WPAFB, Ohio, 1962.

¹³Pierce, F. T., J. Text. Inst., 1937, 28, T45.

¹⁴Kemp, A. J. Text. Inst., 1958, 49 T44.

¹⁵Haas, R., and Dietzius A., The Stretching of the Fabric and the Deformation of the Envelope in Nonrigid Balloons, Springer Verlag, Berlin 1912: Darrow, K. K., NASA Scientific & Technical Information Facility, Translation 1917.

¹⁶Popper, P. G., Cirteria for Rupture of Certain Textile Structures Under Biaxial Stress, Technical Documentary Report, ASD-TDR-62-613, USAF, WPAFB, Ohio, 1962.

The mechanisms involved in the deformation of a biaxially-stressed fabric include:¹⁷

1. Crimp interchange
2. Change in angle between yarns (thread shear)
3. Yarn bending
4. Yarn flattening
5. Yarn extension
6. Friction between filaments
7. Friction between yarns at crossovers
8. Yarn nesting at yarn crossovers
9. Yarn swelling
10. Yarn and fabric rupture

The loading sequence, loading rate, and load distribution (uniformity of stress distribution) also influence the deformation of a fabric.

An analysis of a biaxially-stressed woven fabric that includes all of the parameters, deformation mechanisms and their interrelationships, if it were possible to obtain, would no doubt be so unwieldy that it would be of no practical use to a design engineer. Therefore, only the parameters which are found to exert the greatest influence on the biaxial stress-strain behavior of the fabric will be first considered.

In the program currently in progress mathematical models describing the stress-strain behavior for three fabric geometries have been formulated and solutions obtained for some special cases. The three geometries are illustrated on Figure 16 and include circular cross-section first proposed by Pierce,¹⁸ racetrack cross-section proposed by Kemp¹⁹ and Fabric Research Laboratories,²⁰ and the lenticular cross-section proposed by Fabric Research Laboratories.²¹ The lenticular cross-section is similar to but flatter than the elliptical cross-section yarns proposed by Kemp.

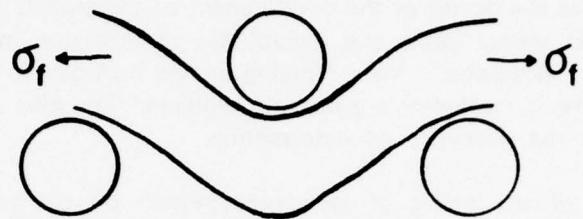
¹⁷ Popper, P. G., A Theoretical Investigation of Crimp Interchange in a Woven Fabric Under Biaxial Stress, Technical Documentary Report, ASD-TDR-62-457, USAF, WPAFB, Ohio, 1962.

¹⁸ Pierce, F. T., J. Text. Inst., 1937, 28, T45.

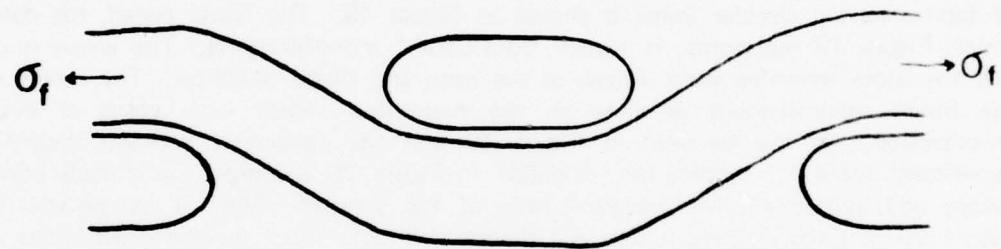
¹⁹ Kemp, A., J. Text. Inst., 1958, 49, T44.

²⁰ Freeston, W. D., Schoppee, M. M., Wall, M. A., Stress-Strain Response of Fabrics Under Two-Dimensional Loading, Part 1. Race-Track Yarn Cross-Section, Fabric Research Laboratories, Dedham, Massachusetts, and the US Army Natick Laboratories, Natick, Massachusetts, Technical Report 73-24-GP, 1971.

²¹ Freeston, W. D., Schoppee, M. M., Wall, M. A., Stress-Strain Response of Fabrics Under Two-Dimensional Loading. Part 2, Lenticular Yarn Cross-Section, Fabric Research Laboratories, Dedham, Massachusetts, and the US Army Natick Laboratories, Natick, Massachusetts, Technical Report 73-25-GP, 1971.



a. Circular Shape



b. Race Track Shape



c. Lenticular Shape

Fig. 16 Three Different Fabric Models

The assumptions and limitations common to the mathematical models for the three different fabric geometries may be found in the cited references.^{21,22} Also included in these references are the details of the development of the models and the results. The models assume linear elastic yarns and include the deformation mechanisms of crimp interchange and yarn extension. Yarn bending is not included in the elastic sense in that crimp interchange is treated as a geometric problem. The effect of the cross-section shape is included in the geometry of deformation.

A comparison of the effect of yarn cross-section on the relation of predicted deformation and stress due to imposed loads is illustrated in Figure 17 for a 1:1 biaxial stress ratio (sphere) and in Figure 18, a 2:1 biaxial stress ratio (cylinder).

An assessment of the quality of the present state of the art in predicting the biaxial stress-strain behavior can be made by examination of the comparison of theory and experiment shown in Figures 19. The results of biaxial tests performed on the Biaxial Test Machine as compared with the theoretical predictions of the load deformation behavior of fabrics having circular yarns is shown in Figure 19. The fabric tested, the data of which Figure 19 represents, is woven from Saran* monofilaments. The weave is plain and the same monofil were woven in the warp and filling direction. The model with the Saran monofilament is close to the theoretical model with yarns of circular cross-section. It can be seen in Figure 19 that the agreement between theory and experiment for a 1:1 loading ratio is good. In Figure 20, a comparison is made between theory and experiment for a loading ratio of 1:2, warp-to-filling. It can be seen from Figure 20 that the agreement between theory and experiment for the stress in the warp yarn is fair at low extensions but improves at the higher extensions. The agreement between theory and experiment is obviously poor for the stress in the filling direction. This indicates that more work is needed to improve the agreement between theory and experiment. Since bending stiffness of the yarns was not considered in the analytical determinations

²¹ Freeston, W. D., Schoppee, M. M., Wall, M. A., Stress-Strain Response of Fabrics Under Two-Dimensional Loading. Part 2, Lenticular Yarn Cross-Section, Fabric Research Laboratories, Dedham, Massachusetts, and the US Army Natick Laboratories, Natick, Massachusetts, Technical Report 73-25-GP, 1971.

²² Freeston, W. D., Schoppee, M. M., Wall, M. A., Stress-Strain Response of Fabrics Under Two-Dimensional Loading, Part 1, Racetrack Yarn Cross-Section, Fabric Research Laboratories, Inc., Dedham, Massachusetts, and the US Army Natick Laboratories, Natick, Massachusetts, Technical Report 73-28-GP, 1971.

*A product of Dow Chemical Co., Midland, Michigan.

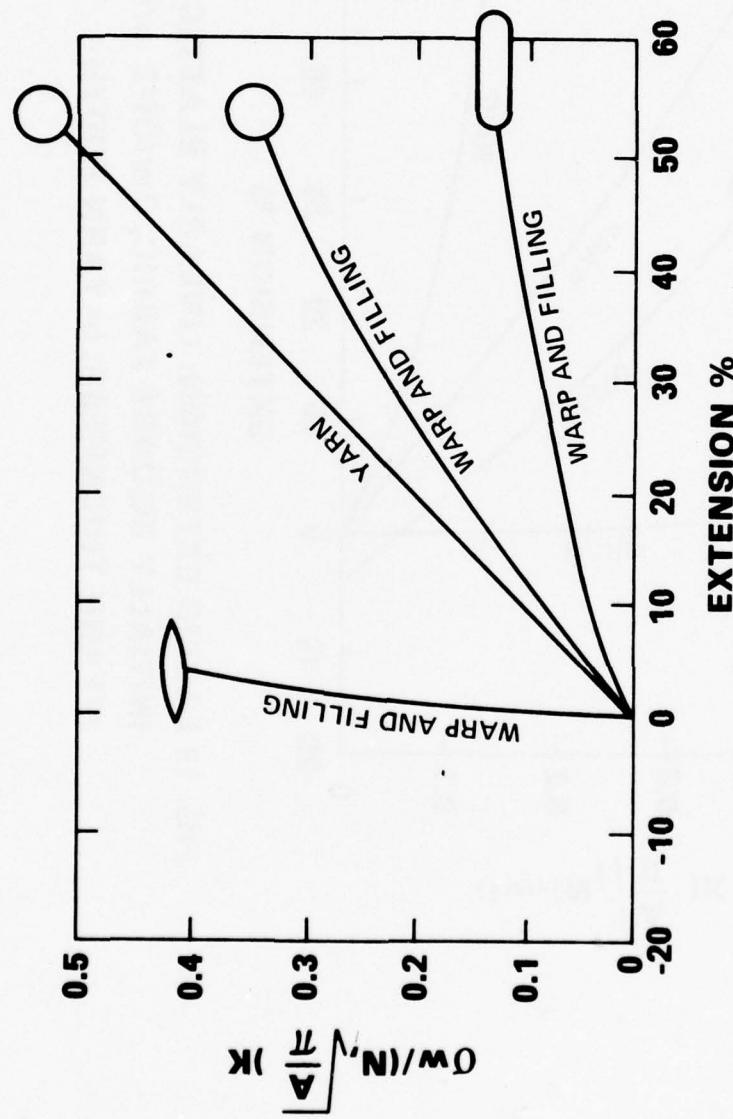


FIG. 17 FABRIC EXTENSION; LINEARLY ELASTIC YARN,
INITIALLY SQUARE FABRIC, $\sigma_w/\sigma_f = 1$, $a/b = 1$ and 10,
a = yarn thickness; b = yarn width

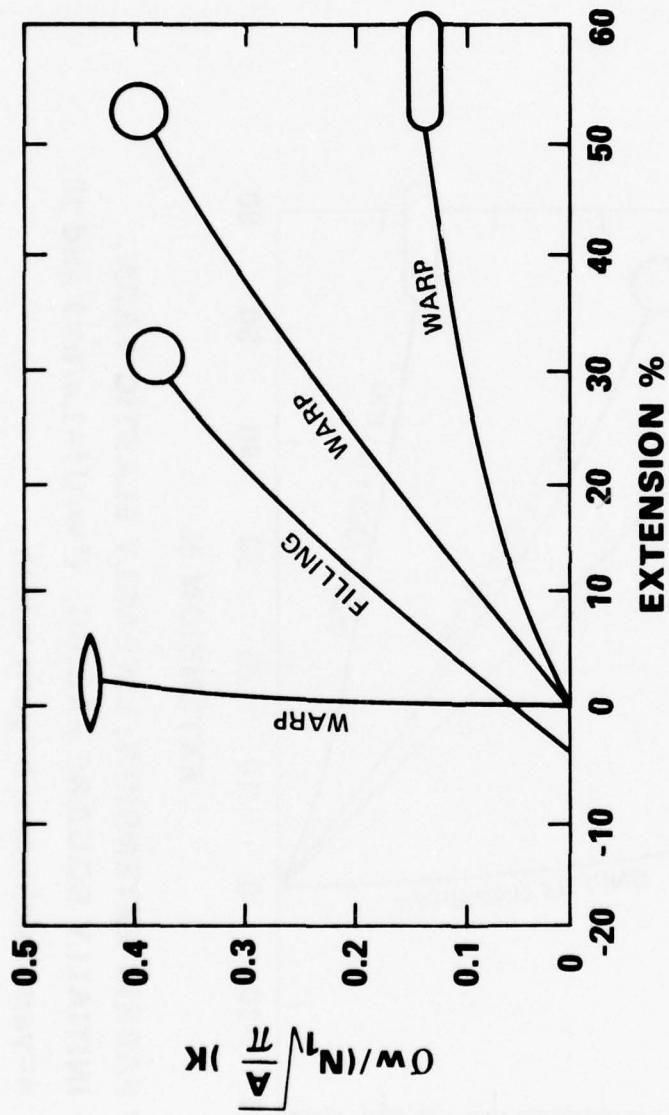


FIG. 18 FABRIC EXTENSION: LINEARLY ELASTIC YARN;
INITIALLY SQUARE FABRIC, $\sigma_w/\sigma_f=2$ $a/b=1$ and 10
a=YARN THICKNESS: b=YARN WIDTH

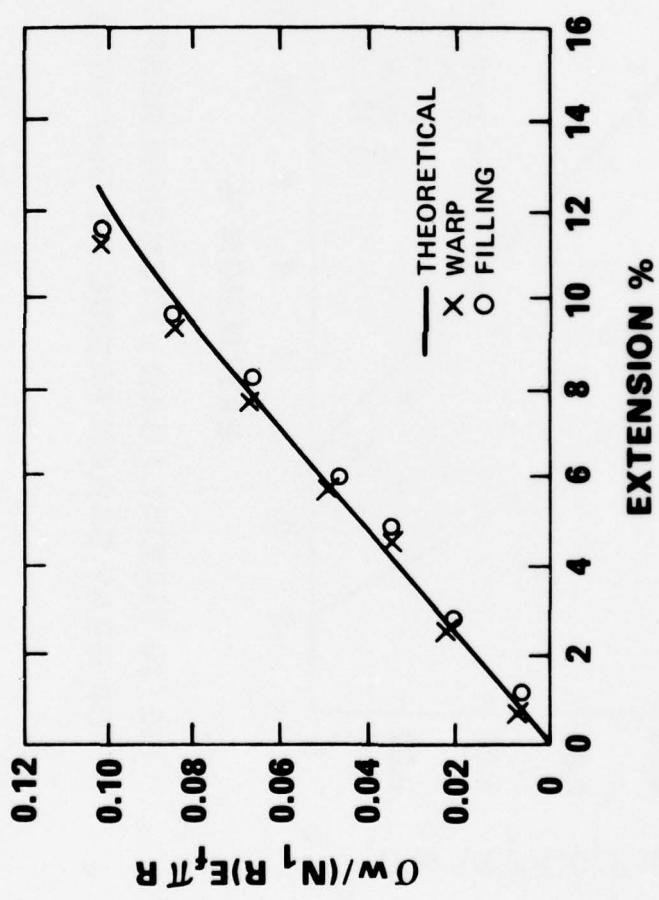


FIG. 19, BIAXIAL LOAD EXTENSION RESPONSE
OF GRAY SARAN FABRIC AT $\sigma_w/\sigma_f = 1$

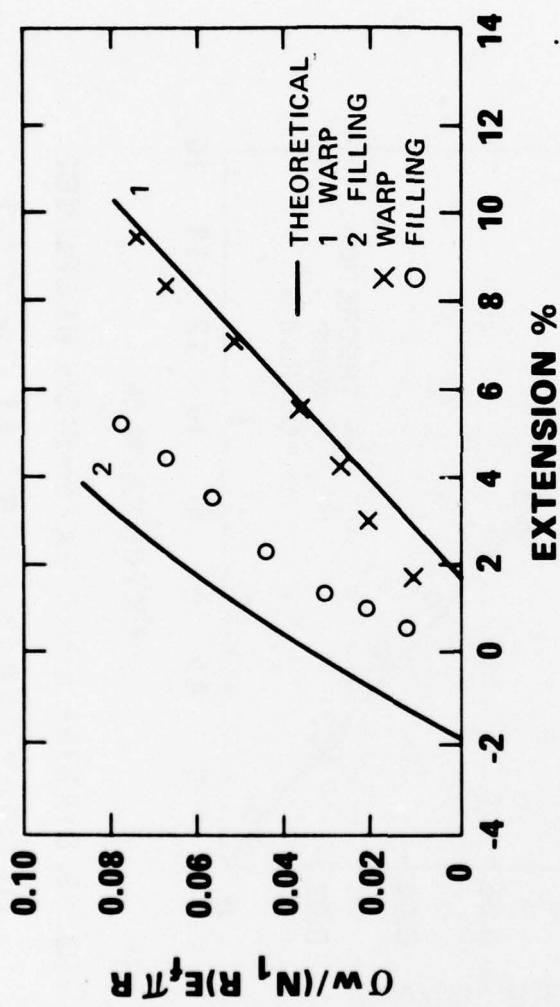


FIG. 20 BIAXIAL LOAD EXTENSION RESPONSE
OF GRAY SARAN FABRIC AT $\sigma_w / \sigma_f = 2$

to date, this is the first avenue of approach to improve the correlation between theory and experiment. Similar comparisons between theory and experiment for racetrack and lenticular cross-section yarns cannot be made because of the lack of experimental evidence. This is another area in which more work needs to be done, and is being done with the support of the U.S. Army Natick Research and Development Command.

Needed Improvements in the State of the Art

A theory to predict the biaxial stress-strain behavior of fabrics has been developed. The theory does not address itself to the problem of predicting the breaking strength of the fabric, it needs to be improved to cover this as well as other aspects of load-elongation behavior of fabrics. The theory needs to be improved to obtain better agreement between the stress-strain curves from theory and experiment. The present theory does not take into account the bending stiffness of the yarn. This deformation mechanism appears to be needed to improve the agreement between theory and experiment particularly at low strains. Also the theory may be improved by including the actual yarn stress-strain behavior in the analysis as opposed to the linear elastic analysis presently used. The theory for yarn of racetrack and lenticular cross-section must be confirmed by experiments on fabrics. Also the theory must be expanded to include fabric models other than the initially square and plain weave. Future fabric models should consider a variation in ends and picks per cm as well as two other basic weaves, the twill and the satin. Finally, a comprehensive computer program which will compute the biaxial stress-strain behavior of fabric of various geometries and material must be written. The output of this program will include such parameters as yarn size and strength and will serve as a basis for the design of the yarn.

Summary

A biaxial stress-strain theory was developed for three fabric models each with yarns of different cross-section shape. It has been established that the cross-sectional shape of the yarn does influence the biaxial stress-strain behavior of the model fabric. The theory for all model fabrics needs to be improved to obtain better agreement with experiment. Further additional experimental confirmation is required for the fabric models with racetrack and lenticular cross-section yarns. A biaxial tensile testing instrument developed by NARADCOM will be used for the projected experimental work.

A computer program is proposed which predicts the stress-strain behavior of fabric, the output of which will serve as a basis for the design of the yarn from which the fabric will be woven.

YARN STRESS-STRAIN CHARACTERISTICS

Present State of the Art

A great deal of work was done on theories which may be used to predict the stress-strain properties of yarns. However, very little theoretical work was done connecting the stress-strain behavior of yarns with the stress-strain behavior of the fabrics in which they are used. This study, with a fabric computer program, relating the maximum stress in a fabric to yarn size and number of yarns per cm, then feeding the information as input to a yarn computer program, which is used to predict the tensile properties of the yarn, forms a natural link between the strength of the yarn and the fabric in which it is to be used. In this section the current state of the art concerning yarn load-elongation behavior will be discussed.

The strength of continuous filament yarns are strongly influenced by twist. As the twist is increased in most continuous filament yarns, its strength decreases, and its weight normally increases for a given strength. Conversely in a continuous filament yarn with no twist, there is not sufficient cohesion among the filaments to mutually support each other to realize the full strength of the yarn. Some twist in continuous filament yarn is therefore essential to achieve operating efficiency in textile processing. The trade-off here is to have sufficient twist in the yarn to achieve the desired mechanical objective with a minimum increase in weight and a minimum decrease in strength.

Again, an experimental approach to the problem of minimum weight, maximum strength yarns, while not as extensive as that of fabrics, is still complicated, time consuming and costly. The theoretical approach to establish yarn strength is the most economical way to approach the problem of yarn stress-strain behavior. Since the theoretical approach which follows includes the fiber stress-strain curve without modification, the theory will provide a useful tool to accelerate the future research on yarns and possibly new fibers to match the mechanical response of textile items and structures not heretofore considered.

Mathematical Modeling: The earliest theory, using the techniques of applied mechanics, to predict the stress-strain behavior of yarns subjected to imposed loads was published by Gegauff in 1907;²³ this work was followed by Pierce in 1937 and 1947,^{24,25} by

²³ Gegauff, C. M., Force et Elasticite des Files en Coton; Bulletin de la Societe Industrielle de Mulhouse: d'Avril 1907.

²⁴ Pierce, T. T., J. Text. Inst., 1937, 28, T45.

²⁵ Pierce, F. T., Text. Res. J., 1947, 17, 123.

Hamburger and Platt in 1949 and 1950 and throughout the early 1950's,^{26,27} and by Hearle in 1958 and through the early 1960's.^{28,29,30} All of the theoretical methods of stress mechanics used up to this time can be described as the force-stress method of applied mechanics. Treloar in 1963 proposed the use of the energy method of stress mechanics.³¹ Riding and Wilson provided experimental verification of Treloar's theoretical approach for most available commercial fibers.³² Hearle in 1969 developed an energy method theory of yarn mechanics equivalent to that of Treloar's.³³ Hearle's theory was reduced to four equations which are easily adapted to computer programming. In fact, Konopasek did develop a computer program for Hearle's theory;³⁴ the full derivation of the energy theory for predicting yarn strength and the programming of the theory for the computer will be found in the cited references.

The following discussion will be confined to listing the assumptions made relative to the yarn model and the agreement found between theory and experiment relative to the yarn stress-strain curve.

The geometric structure in the unstrained state corresponds to that assumed by Platt and Hearle.^{35,36}

²⁶ Hamburger, W. J., J. Text. Inst., 1949, 40, P700.

²⁷ Platt, M. M., Text. Res. J., 1950, 20, 1.

²⁸ Hearle, J. W. S., J. Text. Inst., 1958, 48, T389.

²⁹ Hearle, J. W. S., El-Behery, H. M. A. E., Thakur, V. M., J. Text. Inst. 1960, 51, T299.

³⁰ Hearle, J. W. S., and Thakur, V. M., J. Text. Inst., 1961, 52, T49.

³¹ Treloar, L. R. G., and Riding, G., J. Text. Inst., 1963, 54, T156.

³² Riding, G., and Wilson, J., J. Text. Inst., 1965, T205.

³³ Hearle, J. W. S., J. Text. Inst., 1969, 60, 95.

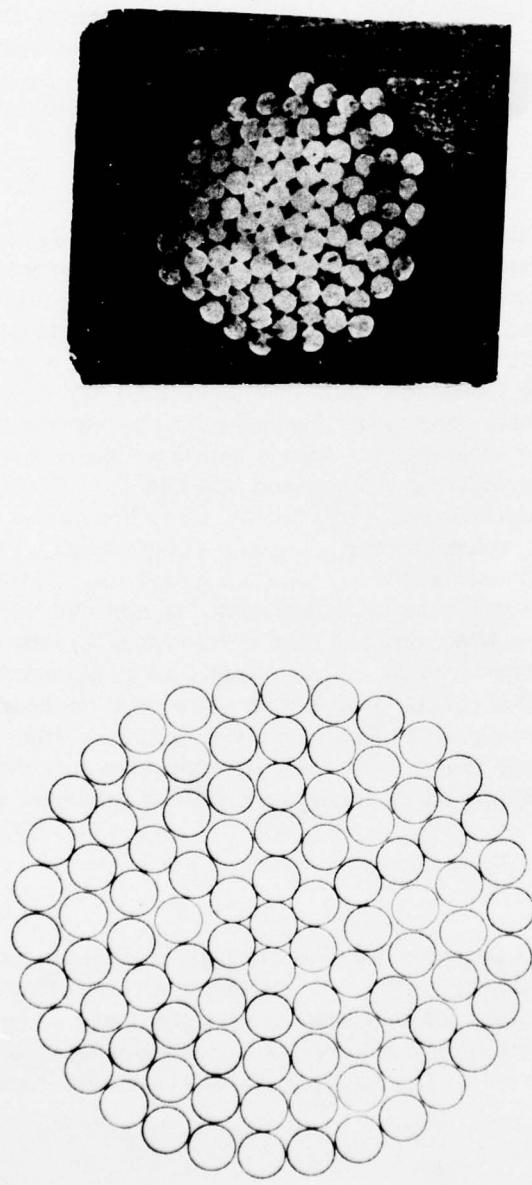
³⁴ Konopasek, M., Univ. of Manch — Inst. of Scien. and Tech., 1970, PH.D. Thesis.

³⁵ Platt, M. M., Text. Res. J., 1950, 20, 1.

³⁶ Hearle, J. W. S., J. Text. Inst., 1958, 48, T389.

- a. The yarn consists of a system of coaxial helices, all having the same number of turns per unit length.
- b. The packing is assumed to be uniform in the sense that the amount of material per unit volume of any small element is the same at all points within the yarn.
- c. The filament diameter is very small when compared with the diameter of the yarn. The filaments are circular in cross-section.
- d. The deformation of the filaments is assumed to take place without change in volume; i.e., the filaments are assumed to be incompressible with respect to hydrostatic pressure.
- e. The deformation of the yarn is assumed to take place without a change in yarn volume; i.e., the axial extension of the yarn is accompanied by a contraction of the yarn radius. For small strains only, the assumption of constant volume is equivalent to a Poisson's ratio of 0.5.
- f. The stress-strain properties of the material which are introduced into the theory are, in general, taken to correspond to the whole stress-strain curve of the material as found from experiments either on single filaments or on the untwisted yarn. Unlike earlier theories, the present theory makes use of the precise form of the whole stress-strain curve up to the breaking extension.
- g. The analysis of the mechanics of the system is carried out by making use of the principle of energy. This involves converting the original stress-strain relation for the material into an energy-strain relation. From this the energy per unit volume of material at any radial position in the yarn may be obtained. The total energy of the yarn in the strained state is then found by integration with respect to radial position. Finally, the axial force is obtained by integration with respect to radial position. Finally, the axial force is obtained by equating the work done in a further small axial strain to the change in the energy of the system. Results using this theory are discussed subsequently in relation to the agreement between theory and experiment.

An experimental yarn was especially made to match the configuration and assumptions of the theoretical model yarn. The yarn was twisted on a laboratory twister using 91 nylon monofilaments, Figure 21. The nylon monofil was tested on the Instron tensile testing machine both as a single and 91 monofil yarn with no twist. The stress-strain curves for the individual nylon monofil were too variable and required a large number of tests to establish a representative curve. The test results for the untwisted 91 filament yarn were found to be in agreement test after test. The test data, from this latter yarn test were stored in the computer to be recalled for the computation of the twisted yarn stress-strain curve.



Mathematical Model Yarn Actual Yarn

Fig. 21 Cross Sections of the Mathematical Model and Actual Yarn.

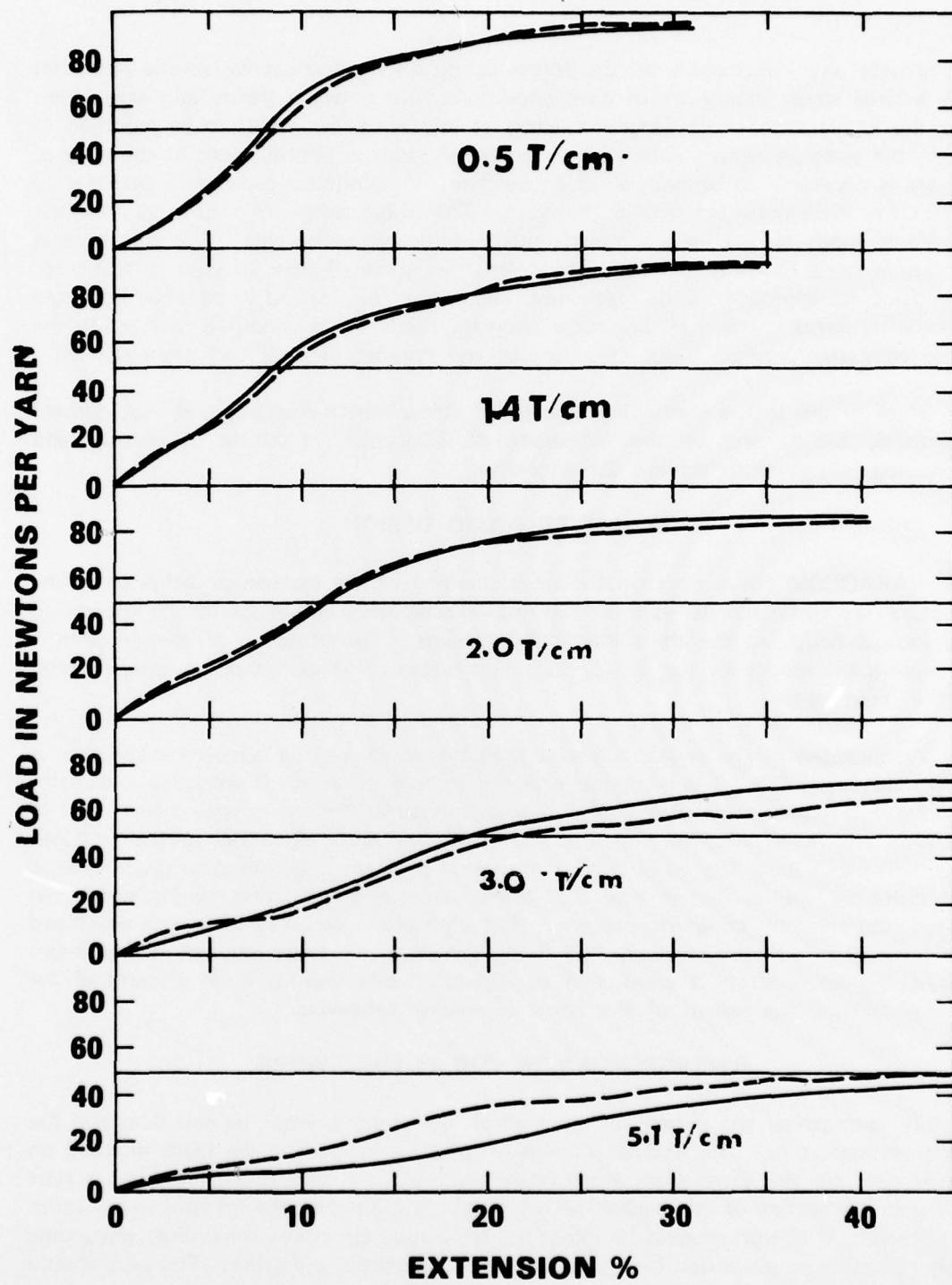
The comparison between the theoretically derived stress-strain curves and the stress-strain curves obtained experimentally is shown on Figure 22 for five yarn twist conditions. Each dashed curve represents an average of 20 tests. It is obvious from the curves that at very low twists (0.5 T/cm), the theoretical and experimental curves are in good agreement. In both cases, the filaments are all parallel or nearly parallel with the yarn axis. Therefore, close agreement is expected for two yarn systems so nearly alike. The theoretical curves also agree closely with experiment except at small values of strain and high values of twist. This indicates that as the twist is increased other factors such as increasing helix angle, variable Poisson's ratio, and loss in fiber strength at high twist are influencing the yarn stress-strain curve in a way which is not accounted for in the theory. All these factors must be studied further and related to the present theory to improve the accuracy of the theoretical prediction of the yarn stress-strain curve.

Needed Improvements in the State of the Art

Although the energy method of stress analysis shows a good fit in the stress-strain curve predicted by theory and that found by experiment for continuous filament yarns, there is a somewhat lack of fit between theory and experiment as the twist in the yarn increases. The theory must be modified to show better agreement in the tensile behavior of yarn between theory and experiment for a wider range of yarn twists than is currently obtained. Such factors as increasing elongation with increasing twist for nylon and polyester is not accounted for in the theory and some adjustment must be made to achieve better agreement between theory and experiment. Also a constant Poisson's ratio is assumed for an extending yarn. This must be checked and adjusted for. Finally, the fibers in the yarn may lose strength and elongation due to the mechanical action of the twisting process alone, and this factor would be most prevalent at high twists. The loss in fiber strength due to twisting must also be checked and accounted for. Finally the stress theory for continuous filament yarns must be expanded to include the prediction of the stress-strain behavior of spun staple fiber yarn, as a start, other factors are introduced, such as fiber staple length and interfiber friction. These factors must be investigated and their relative influence on the strength of spun staple fiber yarns must be determined and included in the theory. Again a computer program for yarns must be written which takes as input from the fabric computer program the yarn strength, yarn size and ends and picks per cm, and provides as output an optimum yarn size for minimum weight and optimum twist.

Summary

Of the two stress analysis theories proposed for predicting the stress-strain behavior of yarns, the force method and the energy method of analysis, the energy method of analysis proved to be the most satisfactory for the whole stress-strain curve. The force method of analysis, while providing more information on the forces acting on an element in the yarn, is limited to small strains only. The energy method on the other hand does



**FIG. 22 LOAD EXTENSION CURVES OF MODEL YARNS
FOR 5 DIFFERENT TWISTS, YARN LINEAL DENSITY
202 MILLIGRAMS/METER**

— THEORETICAL; - - - EXPERIMENTAL.

not provide any information on the forces acting on an element within the yarn, but it is a large strain theory which gives good agreement between theory and experiment over the whole stress-strain curve for yarns at low twist, but needs to be modified to obtain the same agreement with yarns of increasing twist. Improvement in the state of the art is necessary to accomplish this objective. A computer program is available to predict the stress-strain behavior of the yarn. The whole stress-strain curve of the fiber is used as input to the yarn computer program to accomplish this. This will serve as the future basis to tie in the complete stress-strain curve of fiber to yarn to fabric for a fundamental approach to the mechanical behavior of fabrics and yarns when subjected to external stress. The yarn computer program needs to be modified to react to the fabric computer program input of yarn size and number of ends and picks per cm.

Work in this line is continuing at the U.S. Army Natick Research and Development Command, U.S.A. and at the University of Manchester, Institute of Science and Technology, England to achieve this objective.

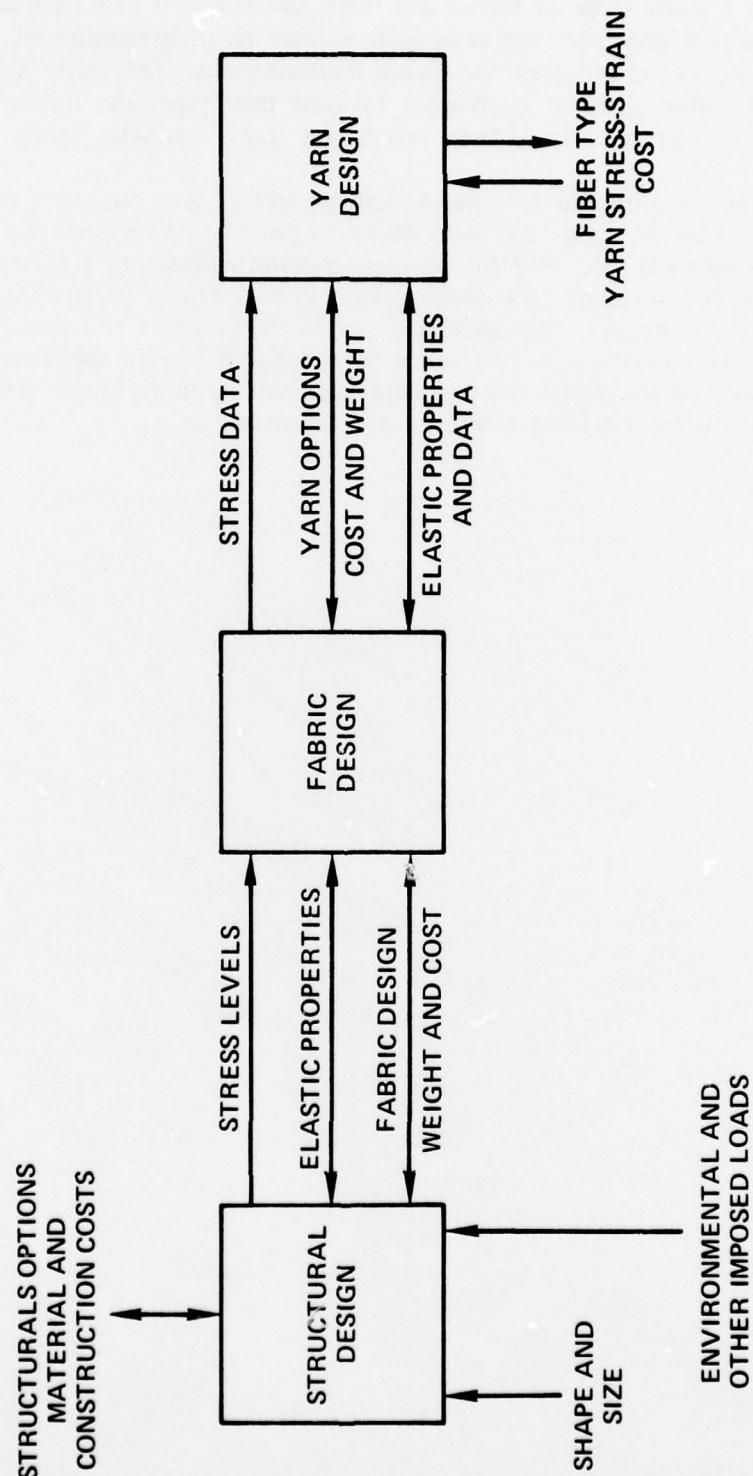
COMPUTER AIDED DESIGN

NARADCOM has a start on the understanding of the mechanical behavior of the structure, the fabric and the yarn but has not yet established any model for the interaction and mutual design of the three interacting systems. The objective of this program is to develop the model for the above interacting systems and work out the mutual design of the three items.

An overview of the entire computer program, which will be achieved eventually, is shown on Figure 23. The computer program for the structures is workable and yields aerodynamic loads of lift, drag and moments as well as the fabric weight and loads in the warp and filling direction which can feed into the fabric computer program. When it is fully developed, the output from the fabric program is expected to give the ends and picks per cm, as well as warp and filling yarn stress and fabric weight, which can be fed into the yarn computer program. The yarn computer program is well developed and provides information on warp and filling yarn size, the twist required for minimum weight, maximum strength yarns, and an adjusted fabric weight, if, as a result of the yarn properties, the weight of the fabric is revised downward.

RECOMMENDATION FOR FUTURE WORK

To summarize, the design for mechanical fabrics starts with the end item and the forces acting on it. The analysis of these forces are related to the loads imposed on the fabrics and the yarns from which they are made. NARADCOM has made a start on the understanding of the mechanical behavior of a limited number of structures, fabrics and fibers. This work should be expanded to include structures other than pneumatic shelters; i.e. frame supported tents, pressure suits, parachutes, and others. The work should



**FIG. 23 OPTIMIZATION OF STRUCTURE DESIGN AND ITS
INTERACTION WITH FABRIC AND YARN DESIGN**

be expanded to include a wider range of fabrics and fabric constructions than have been considered to date. Considerable work has been accomplished on understanding of the mechanical behavior of nylon and polyester continuous filament yarns. This work should be expanded to include other types of continuous filament fiber types and further to include staple fiber yarns such as cotton, linen, wool, and staple synthetic fibers.

A computer program is projected to cover structures, fabrics, and yarns and their interrelationships to optimize the fabric and yarn design to provide a functional reliable structure of minimum weight and cost. With the necessary theories validated by experiment and the computer program completed, an engineer need only to initiate a design concept by giving as input to the computer, the shape and size of the structure and the wind speed the structure will have to withstand. The computer, within minutes, will then provide data on aerodynamic loads on the shelter and complete specifications on the fabrics, yarns, and fibers without the engineers tracing a line on a piece of paper.

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